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Effects of Coriolis force and centrifugal force on acoustic waves propagating along the surface of a piezoelectric half-space

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Abstract. This article presents an analysis on the effect of rotation upon surface acoustic waves propagating in a piezoelectric half-space. It shows that the Rayleigh wave and the Bleustein-Gulyaev wave may be suppressed by rotation, depending on the physical properties of materials, and that these surface waves, if they exist, are generally dispersive. The effects of the Coriolis force and the centrifugal force on dispersion are generally of the same order. Some numerical results generated using PZT-5H as the model material are included for illustration.

Keywords. Surface acoustic waves, dispersion, piezoelectric crystals, gyroscopes.

1. Introduction

There have been many recent developments on designing and fabricating micro gyros (i.e., rotation rate sensors), see Greiff, et al [1], Söderkvist [2], Bernstein, et al [3], Tanaka, et al [4], and Maenaka, et al [5], for example. The key component of a vibratory micro gyro is a micromachined piezoelectric structure, such as a cantilever beam, that vibrates at its resonance driven by an applied electric field. This primary vibration interacts with rotation, resulting in a secondary vibration due to the effect of both Coriolis force and centrifugal force, and the magnitude of the resulting secondary motion depends upon the rotation rate. The extremely high sensitivity of these gyros leads to very high theoretical resolution, but it also causes these devices very sensitive to the operating environment. For instance, they usually suffer very low mechanical quality factors due to the air damping effect. Some researchers, such as Kurosawa, et al [6], have been developing surface acoustic wave micro gyros based on the phenomenon that surface acoustic waves are disturbed by rotation due to the presence of Coriolis force and centrifugal force. This requires fabricating interdigital transducers onto surfaces of piezoelec-

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tric materials using the IC technology, for generating and detecting acoustic waves propagating along the surfaces of these piezoelectric materials.

The rotation effect on acoustic waves propagating along the surface of an elastic, isotropic, half-space has been studied by Wren and Burdess [7] and Clarke and Burdess [8]. There is little literature on analysis of the rotation-disturbed acoustic waves propagating along the surface of a piezoelectric half-space. We present in the next section a mathematical formulation that governs surface acoustic waves propagating in a piezoelectric half-space in rotation. It is seen from the displacement equations of motion that the Coriolis force effect on the wave *magnitude* is of the first order and the centrifugal force effect is of the second. Our analysis presented in Section 3 shows, however, that the effects of the Coriolis force and the centrifugal force on the surface acoustic wave speed are of the same order. It is known that solutions of surface acoustic waves in piezoelectric materials are not unique, and that there exist generally two types of surfaces waves in piezoelectric solids, i.e., the Rayleigh wave (Rayleigh [9]) and the Bleustein-Gulyaev wave (Bleustein [10], Gulyaev [11]). Some numerical results for a model material, PZT-5H, are presented in Section 4 to illustrate the effects of the Coriolis force and the centrifugal force on the Rayleigh wave. Our analysis suggests that rotation suppress the Bleustein-Gulyaev wave propagating in our model material, at least when the rotation rate is very small in comparison with the wave frequency. We present in Section 5 our discussion on the existence of surface wave solutions when the rotation rate is very small comparing to the wave frequency, using the standard perturbation analysis. On this issue, we particularly note the early work of Lothe and Barnett [12] on the existence and uniqueness of surface wave solutions propagating in non-rotating piezoelectric solids.

2. Basic equations

We consider a linear piezoelectric body that occupies the half space, denoted by $x_2 \leq 0$ in the Cartesian frame shown in Figure 1. To study the rotation effect, we let the body rotate about the x_2-axis at a constant rate Ω. In the frame that rotates with the body, we write the equations of motion and Coulomb's law as follows:

Equations of Motion:

$$\nabla \cdot \mathbf{T} - 2\rho\Omega\mathbf{j} \times \dot{\mathbf{u}} - \rho\Omega^2\mathbf{j} \times [\mathbf{j} \times (\mathbf{x} + \mathbf{u})] = \rho\ddot{\mathbf{u}} \tag{1}$$

Coulomb's Law:

$$\nabla \cdot \mathbf{D} = 0 \tag{2}$$

Here, $\mathbf{T}$, $\mathbf{u}$, $\mathbf{D}$ and ρ are the stress tensor, the displacement vector, the dielectric displacement vector, and the mass density, respectively; $\mathbf{x}$ denotes the position

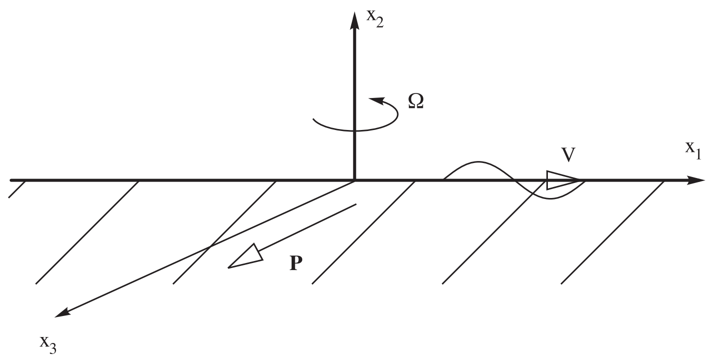

Figure 1. Schematic of a piezoelectric half space rotating about x_2-axis

vector; and $\mathbf{j}$ stands for the unit vector along the x_2-axis. We denote by ∇ the gradient operator in 3-d space and by a superimposed dot differentiation with respect to the time parameter t. Note that $-2\rho\Omega\mathbf{j}\times\dot{\mathbf{u}}$ and $-\rho\Omega^2\mathbf{j}\times[\mathbf{j}\times(\mathbf{x}+\mathbf{u})]$ are, respectively, the Coriolis force and the centrifugal force. For linear piezoelectric materials, we have the following constitutive relations:

$$\mathbf{T} = \mathbf{c}\ \mathbf{S} - \mathbf{e}^T\mathbf{E} \tag{3}$$

$$\mathbf{D} = \mathbf{e}\ \mathbf{S} + \epsilon\mathbf{E} \tag{4}$$

where $\mathbf{c}$, $\mathbf{e}$ and ϵ are the elasticity tensor, the piezoelectric tensor, and the dielectric tensor, respectively. The superscript T indicates transpose. The strain tensor $\mathbf{S}$ and the electric field intensity vector $\mathbf{E}$ are related to the displacement vector $\mathbf{u}$ and the electric potential ϕ through the following:

$$\mathbf{S} = \frac{1}{2}[\nabla\mathbf{u} + (\nabla\mathbf{u})^T] \tag{5}$$

$$\mathbf{E} = -\nabla\phi \tag{6}$$

We are interested in waves propagating along a surface that is electroded and free of mechanical loads, and hence we consider the following boundary conditions:

$$\mathbf{T}\cdot\mathbf{j} = \mathbf{0},\ x_2 = 0 \tag{7}$$

$$\phi = 0,\ x_2 = 0 \tag{8}$$

Because of the linearity of the formulation, we can decompose the displacement: $\mathbf{u}(\mathbf{x}, t) = \mathbf{u}^s(\mathbf{x}) + \mathbf{u}^w(\mathbf{x}, t)$; here $\mathbf{u}^s$ stands for the static deformation caused by rotation and measured in the rotating frame, and $\mathbf{u}^w$ is the wave solution superimposed upon the static deformation. Using this decomoposition, we can derive from

Equations (1)-(8) the governing equations for $\mathbf{u}^w$. We note that these equations remain the same except the equations of motion (1) which is given as follows:

$$\nabla \cdot \mathbf{T} - 2\rho\Omega\mathbf{j} \times \dot{\mathbf{u}} - \rho\Omega^2\mathbf{j} \times (\mathbf{j} \times \mathbf{u}) = \rho\ddot{\mathbf{u}} \tag{9}$$

In Equation (9) and hereafter, we denote the wave solution by $\mathbf{u}$ instead of $\mathbf{u}^w$ for convenience. This should not result in any confusion because we would only discuss the wave solution from now on.

To seek surface wave solutions, we require that both the displacement and electric potential diminish with depth, i.e.

$$\lim_{x_2 \longrightarrow -\infty} \mathbf{u} = \mathbf{0}, \quad \lim_{x_2 \longrightarrow -\infty} \phi = 0 \tag{10}$$

We now specialize the constitutive relations to piezoelectric crystals of a tetragonal system with point group $4mm$, and we note that the polycrystalline piezoelectric ceramics are of the same symmetry. Placing the x_3 axis along the four-fold axis, and using the compressed matrix notation (Tiersten [14]), i.e.,

$$[\mathbf{T}] = [T_{11}, T_{22}, T_{33}, T_{23}, T_{31}, T_{12}]^T \tag{11}$$

$$[\mathbf{S}] = [S_{11}, S_{22}, S_{33}, 2S_{23}, 2S_{31}, 2S_{12}]^T \tag{12}$$

$$[\mathbf{D}] = [D_1, D_2, D_3]^T \ , \ [\mathbf{E}] = [E_1, E_2, E_3]^T \tag{13}$$

and

$$[\mathbf{u}] = [u_1, u_2, u_3]^T \ , \ [\mathbf{x}] = [x_1, x_2, x_3]^T \tag{14}$$

one can write the elasticity tensor, the piezoelectric tensor and the dielectric tensor by the following matrices:

$$[\mathbf{c}] = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \tag{15}$$

$$[\mathbf{e}] = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{13} & e_{13} & e_{33} & 0 & 0 & 0 \end{bmatrix} \tag{16}$$

$$[\epsilon] = \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{11} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix} \tag{17}$$

We now consider a wave propagating along the x_1-direction, and seek solutions of the form

$$\mathbf{u}(\mathbf{r},t) = \mathbf{A}(x_2)e^{i(kx_1-\omega t)}\ ,\ \phi(\mathbf{r},t) = A_4(x_2)e^{i(kx_1-\omega t)} \tag{18}$$

where $k > 0$ and $\omega > 0$ are the wave number and the frequency, respectively, and they are related to the phase velocity V by $V = \omega/k$. For convenience, we introduce a new variable ψ defined by

$$\psi = \phi - \frac{e_{15}}{\epsilon_{11}}u_3 \tag{19}$$

This leads to the following representaton of the equations of motion (9) and Coulomb's law (2) in terms of displacements $u_k (k = 1, 2, 3)$ and the function ψ:

$$c_{11}u_{1,11} + c_{12}u_{2,21} + c_{66}(u_{1,22} + u_{2,12}) = \rho\ddot{u}_1 + 2\rho\Omega\dot{u}_3 - \rho\Omega^2u_1 \tag{20}$$

$$c_{66}(u_{1,21} + u_{2,11}) + c_{12}u_{1,12} + c_{11}u_{2,22} = \rho\ddot{u}_2 \tag{21}$$

$$\bar{c}_{44}(u_{3,11} + u_{3,22}) = \rho\ddot{u}_3 - 2\rho\Omega\dot{u}_1 - \rho\Omega^2u_3 \tag{22}$$

$$\psi_{,11} + \psi_{,22} = 0 \tag{23}$$

Correspondingly, the boundary conditions (7) and (8) become

$$x_2 = 0:\ T_6 = c_{66}(u_{1,2} + u_{2,1}) = 0 \tag{24}$$

$$x_2 = 0:\ T_2 = c_{12}u_{1,1} + c_{11}u_{2,2} = 0 \tag{25}$$

$$x_2 = 0:\ T_4 = \bar{c}_{44}u_{3,2} + e_{15}\psi_{,2} = 0 \tag{26}$$

$$x_2 = 0:\ \phi = \psi + \frac{e_{15}}{\epsilon_{11}}u_3 = 0 \tag{27}$$

where

$$\bar{c}_{44} = c_{44} + \frac{e_{15}^2}{\epsilon_{11}}\ ,\ c_{66} = \frac{c_{11} - c_{12}}{2} \tag{28}$$

We note from the displacement equations of motion (20) - (23) that the effect of the centrifugal force on the particle displacements is one order higher than that of the Coriolis force if the rotation rate Ω is much smaller than the wave frequency ω. Our analysis presented in the next section shows, however, that their effects on the wave speed are of the same order.

3. Surface wave solutions

To satisfy the conditions in (10), we consider solutions of the following form:

$$u_1 = A_1 e^{k\eta x_2} e^{i(kx_1 - \omega t)} \tag{29}$$

$$u_2 = iA_2 e^{k\eta x_2} e^{i(kx_1 - \omega t)} \tag{30}$$

$$u_3 = iA_3 e^{k\eta x_2} e^{i(kx_1 - \omega t)} \tag{31}$$

$$\psi = iA_4 e^{kx_2} e^{i(kx_1 - \omega t)} \tag{32}$$

where $i = \sqrt{-1}$ is the unit imaginary number. We require $\eta > 0$ so that the conditions in (10) are satisfied. We note that Equation (23) is automatically satisfied by ψ given in (32). Substitution of Equations (29)-(31) into Equations (20)-(22) leads to the following eigenvalue problem:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & 0 \\ a_{31} & 0 & a_{33} \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \mathbf{0} \tag{33}$$

where

$$a_{11} = V_2^2\eta^2 - V_1^2 + V^2(1 + \frac{\Omega^2}{\omega^2}) \ , \ a_{12} = -(V_1^2 - V_2^2)\eta \ , \ a_{13} = a_{31} = -2V^2\frac{\Omega}{\omega}$$

$$a_{21} = (V_2^2 - V_1^2)\eta \ , \ a_{22} = V_1^2\eta^2 - V_2^2 + V^2 \ , \ a_{33} = V_3^2(\eta^2 - 1) + V^2(1 + \frac{\Omega^2}{\omega^2})$$

Here the longitudinal wave speed V_1 and the shear wave speeds V_2 and V_3 are defined by

$$V_1 = \sqrt{c_{11}/\rho}, \ V_2 = \sqrt{c_{66}/\rho}, \ V_3 = \sqrt{\bar{c}_{44}/\rho} \tag{34}$$

For (33) to have non-zero solutions, the determinant of the coefficient matrix must vanish

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & 0 \\ a_{31} & 0 & a_{33} \end{vmatrix} = 0 \tag{35}$$

This leads to

$$[(\eta^2 - 1)V_1^2 + V^2][(\eta^2 - 1)V_2^2 + V^2][(\eta^2 - 1)V_3^2 + V^2]$$

$$= 4V^4[\eta^2 V_1^2 - V_2^2 + V^2](\frac{\Omega}{\omega})^2$$

$$-V^2[\eta^2 V_1^2 - V_2^2 + V^2]\{[(\eta^2 - 1)V_3^2 + V^2] + [\eta^2 V_2^2 - V_1^2 + V^2] + V^2(\frac{\Omega}{\omega})^2\}\,(\frac{\Omega}{\omega})^2 \tag{36}$$

This relation indicates that the wave speed V generally depends upon the frequency ω unless $\Omega = 0$, i.e., the surface waves are generally dispersive due to the rotation effect. We note that on the right-hand side of the dispersion relation (36) the first term results from the Coriolis force and the second term is due to the centrifugal force, and that they are of the same order in terms of the ratio Ω/ω which is a small parameter. To satisfy the boundary conditions (24)-(27), we need three independent eigenvectors and this requires that the dispersion relation has three real roots.

We note that (36) can be viewed as a cubic algebraic equation on the eigenvalue η. The coefficients of this algebraic equation are all real although they are very complicated. To avoid the algebraic complexity, we consider the following general form of cubic algebraic equation:

$$ax^3 + bx^2 + cx + d = 0 \tag{37}$$

We recall [13] that this equation has three distinct real roots if its coefficients are all real and satisfy the following inequality:

$$(2b^3/27 - cb/3 + d)^2 + \frac{4}{9}(c - b^2/3)^3 < 0 \tag{38}$$

We note that the coefficients of (36) involve the bulk wave speeds which are material constants, the ratio of the wave frequency over the rotation rate and the unknown surface wave speed. Satisfaction of these coefficients to the inequality (38) can not be verified because the surface wave speed is yet to be determined. We here assume that (36) has three distinct positive roots and we later verify this assumption numerically for specific model materials. This assumption leads to the following representation for the wave solutions:

$$u_1 = \sum_{j=1}^{3} C_j A_1^{(j)} e^{k\eta_j x_2} e^{i(kx_1 - \omega t)} \tag{39}$$

$$u_2 = i \sum_{j=1}^{3} C_j A_2^{(j)} e^{k\eta_j x_2} e^{i(kx_1 - \omega t)} \tag{40}$$

$$u_3 = i \sum_{j=1}^{3} C_j A_3^{(j)} e^{k\eta_j x_2} e^{i(kx_1 - \omega t)} \tag{41}$$

$$\psi = iC_4 e^{kx_2} e^{i(kx_1 - \omega t)} \tag{42}$$

Here, $\mathbf{A}^{(j)}$ $(j = 1, 2, 3)$ are eigenvectors corresponding to the eigenvalues η_j^2. The magnitudes C_j $(j = 1, 2, 3, 4)$ are to be determined. Substituting (39)-(42) into the boundary conditions (24)-(27) yields

$$\begin{bmatrix} b_{11} & b_{12} & b_{13} & 0 \\ b_{21} & b_{22} & b_{23} & 0 \\ \bar{c}_{44}A_3^{(1)}\eta_1 & \bar{c}_{44}A_3^{(2)}\eta_2 & \bar{c}_{44}A_3^{(3)}\eta_3 & e_{15} \\ e_{15}A_3^{(1)} & e_{15}A_3^{(2)} & e_{15}A_3^{(3)} & \epsilon_{11} \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \mathbf{0} \qquad (43)$$

where $b_{1j} = A_1^{(j)}\eta_j - A_2^{(j)}$, $b_{2j} = c_{12}A_1^{(j)} + c_{11}A_2^{(j)}\eta_j$ $(j = 1, 2, 3)$. For nontrivial solutions, the determinant of the coefficient matrix must vanish, i.e.

$$\begin{vmatrix} b_{11} & b_{12} & b_{13} & 0 \\ b_{21} & b_{22} & b_{23} & 0 \\ \bar{c}_{44}A_3^{(1)}\eta_1 & \bar{c}_{44}A_3^{(2)}\eta_2 & \bar{c}_{44}A_3^{(3)}\eta_3 & e_{15} \\ e_{15}A_3^{(1)} & e_{15}A_3^{(2)} & e_{15}A_3^{(3)} & \epsilon_{11} \end{vmatrix} = 0 \qquad (44)$$

From (44), we shall evaluate the effect of rotation upon the surface wave speed V.

4. Effects of Coriolis force and centrifugal force

We now study the rotation effect on the phase velocity numerically using PZT-5H as a model material whose relevant properties (Tiersten [14]) are listed below:

$$c_{11} = c_{22} = 1.26 \times 10^{11}(N/m^2)\ ,\ c_{44} = 0.230 \times 10^{11}(N/m^2)$$

$$c_{12} = 0.795 \times 10^{11}(N/m^2)\ ,\ c_{66} = (c_{11} - c_{12})/2$$

$$e_{15} = 17.0(C/m^2)\ ,\ \epsilon_{11} = 1.51 \times 10^{-8}(C/V - m)\ ,\ \rho = 7500(\text{kg/m}^3)$$

In the numerical analysis, we use the following eigenvectors:

$$A_1^{(i)} = 1,\ A_2^{(i)} = -a_{21}/a_{22},\ A_3^{(i)} = -a_{31}/a_{33} \text{ for } i = 1, 2$$

$$A_3^{(3)} = 1,\ A_1^{(3)} = -a_{33}/a_{31},\ A_2^{(3)} = a_{21}a_{33}/(a_{22}a_{31})$$

To determine the dependence of the surface wave speed V upon the rotation rate Ω, we let the rotation rate Ω increase gradually from zero and solve Equations (36) and (44) using an iterative procedure: i) Solve Equation (36) for positive eigenvalues $\eta_i (i = 1, 2, 3)$ for an initial guess of the surface wave speed V_0; ii) Using these eigenvalues, solve Equation (44) for the surface wave speed V; iii) Repeat these steps until a prescribed accuracy is met: $|V - V_0| < \epsilon << 1$. It is evident that the first two equations in (33) are decoupled from the third when $\Omega = 0$: no rotation. This generally leads to two surface wave solutions: one is within the sagittal (x_1-x_2) plane ($u_3 \equiv 0$), called the Rayleigh wave, and the other is perpendicular to the x_1-x_2 plane ($u_1 \equiv 0$ and $u_2 \equiv 0$), called the Bleustein-Gulyaev wave. For $\Omega \neq 0$, all the equations in (33) are coupled and hence the surface waves would have three components. We expect that a very small rotation rate Ω would result in small deviations of the wave speed and the eigenvalues from their respective non-rotation ($\Omega = 0$) values. Hence, we first obtain the non-rotation surface wave solutions which would provide initial values for the iterative procedure outlined above. For our model material, we find, for $\Omega = 0$, the Rayleigh wave speed $\bar{V}_R \approx 1655.56(m/s)$ with the corresponding eigenvalues:

$$\eta_1^2 = 0.8368 \ , \ \eta_2^2 = 0.1157 \ , \ \eta_3^2 = 0.5121$$

and the Bleustein-Gulyaev wave speed $\bar{V}_B = 2369.50(m/s)$ with the corresponding eigenvalues:

$$\eta_1^2 = 0.6658 \ , \ \eta_2^2 = -0.8111 \ , \ \eta_3^2 = 7.138 \times 10^{-4}$$

We note that $\eta_2^2 < 0$ for the Bleustein-Gulyaev wave. This implies the absence of the Bleustein-Gulyaev wave when the body rotates (at least for a small rotation rate Ω). This negative eigenvalue does not prevent us from finding the Bleustein-Gulyaev wave when $\Omega = 0$ because, in this case, Equations (33) are decoupled and the Bleustein-Gulyaev wave corresponds to a single eigenvalue $\eta_3 = 7.138 \times 10^{-4}$. We hence focus our numerical analysis on the rotation effect upon the Rayleigh wave. The dependence of the Rayleigh wave speed V_R upon the scaled rotation rate Ω/ω is plotted in Figure 2, and the approximated solution, resulting from neglecting the centrifugal force, is included for comparison. It is seen that the difference is significant even for small rotation rate. This can be expected from the dispersion relation (36) which indicates that the effect of the centrifugal force on the surface wave speed is of the same order as that of the Coriolis force. There have been proposals to design rotation rate sensors using the dispersive characteristic of the Rayleigh wave due to rotation. Our analysis suggests that the error caused by neglecting the centrifugal force is significant for such applications.

5. Discussion on existence of surface wave solutions

The above analysis indicates the absence of the Bleustein-Gulyaev wave in PZT-5H when the body rotates, at least, with a small rotation rate. This has motivated us to study the existence of surface wave solutions when a piezoelectric body rotates slowly, and some results from a perturbation analysis is presented in this section. In obtaining these results, we have assumed that the surface wave speeds V_R and V_B, if they exist, as well as the corresponding eigenvalues $\eta_i, i = 1, 2, 3$, deviate slightly from their non-rotation values $\bar{V}_R, \bar{V}_B, \bar{\eta}_i, i = 1, 2, 3$, when the rotation rate Ω is very small in comparison with the wave frequency ω, i.e.

$$V_R^2 \approx \bar{V}_R^2 + \alpha_R(\frac{\Omega}{\omega})^2 \ , \ V_B^2 \approx \bar{V}_B^2 + \alpha_B(\frac{\Omega}{\omega})^2, \tag{45}$$

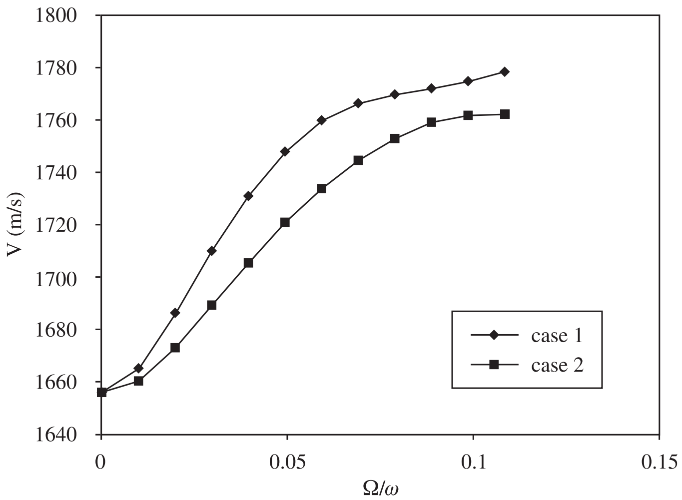

Figure 2. The Rayleigh wave speed versus the rotation ratio $\frac{\Omega}{\omega}$: Case 1: including the centrifugal force; Case 2: neglecting the centrifugal force

$$\eta_i^2 \approx \bar{\eta}_i^2 + \beta_i(\frac{\Omega}{\omega})^2 \ (i = 1, 2, 3) \tag{46}$$

where α_R, α_B and β_i are constants. We note from (36) and (44) the dependence of the wave speeds and eigenvalues upon $(\frac{\Omega}{\omega})^2$.

5.1 Non-rotation surface wave solutions

We turn now to determine the non-rotation values of the surface wave speeds and the eigenvalues. Setting $\Omega = 0$ in (36), we obtain

$$\bar{\eta}_j^2 = 1 - (\frac{V}{V_j})^2, \ j = 1, 2, 3 \tag{47}$$

Noting that the first two equations in (33) become decoupled from the third because $a_{13} = a_{31} = 0$ for $\Omega = 0$, we choose the following eigenvectors:

$$A_1^{(1)} = 1 \ , \ A_2^{(1)} = -\bar{\eta}_1 \ , \ A_3^{(1)} = 0 \text{ for } \ \eta = \bar{\eta}_1 = \sqrt{\bar{\eta}_1^2} \tag{48}$$

$$A_1^{(2)} = -\bar{\eta}_2 \ , \ A_2^{(2)} = 1 \ , \ A_3^{(2)} = 0 \text{ for } \ \eta = \bar{\eta}_2 = \sqrt{\bar{\eta}_2^2} \tag{49}$$

$$A_1^{(3)} = A_2^{(3)} = 0 \ , \ A_3^{(3)} = 1 \text{ for } \eta = \bar{\eta}_3 = \sqrt{\bar{\eta}_3^2} \tag{50}$$

Substitution of (48)-(50) into (44) yields

$$F_1(V^2) = 0, \text{ where } F_1(V^2) = 4V_2^2\bar{\eta}_1\bar{\eta}_2 + (1 + \bar{\eta}_2^2)[V_1^2(1 - \bar{\eta}_1^2) - 2V_2^2] \tag{51}$$

for the Rayleigh wave and/or

$$F_2(V^2) = 0, \text{ where } F_2(V^2) = \epsilon_{11}\bar{c}_{44}\sqrt{1-(\frac{V}{V_3})^2} - e_{15}^2 \tag{52}$$

for the Bleustein-Gulyaev wave.

It is known that (51) has one and only one root within the range $(0, V_2)$. In fact, Lothe and Barnett [12] show that there are at most two subsonic surface waves in this case, which are the Rayleigh wave and the Bleustein-Gulyaev wave. The Bleustein-Gulyaev wave $\bar{V}_B$ can be found explicitly from (52)

$$V = \bar{V}_B = \sqrt{1-(\frac{e_{15}^2}{\epsilon_{11}\bar{c}_{44}})^2}\, V_3 < V_3 \tag{53}$$

We turn now to determine the range of the Rayleigh wave speed $\bar{V}_R$. We note that

$$V_2^2 = \frac{c_{66}}{\rho} = \frac{c_{11}-c_{12}}{2\rho} \le \frac{c_{11}}{2\rho} = \frac{V_1^2}{2}$$

and we obtain from (51)

$$F_1(\frac{2}{3}V_2^2) = \frac{4}{3}V_2^2(\sqrt{3-2(\frac{V_2}{V_1})^2} - \frac{4}{3}) \ge \frac{4}{3}V_2^2(\sqrt{2}-\frac{4}{3}) > 0, \tag{54}$$

and

$$F_1(V_2^2) = -V_2^2 < 0 \tag{55}$$

Thus, (51) must have a root in the range: $(\sqrt{\frac{2}{3}}V_2, V_2)$, and we hence have $\frac{2}{3}V_2^2 < \bar{V}_R^2 < V_2^2$ because the Rayleigh wave speed $\bar{V}_R$ is the only root of (51) within the range $(0, V_2)$.

5.2 Perturbed surface waves by rotation

To obtain the rotation-perturbed surface wave solutions, we substitute the approximated eigenvalues given by (46) into the dispersion relation (36) and this leads to

$$\beta_1 = \frac{\gamma_1}{(\bar{\eta}_1^2-\bar{\eta}_2^2)(\bar{\eta}_1^2-\bar{\eta}_3^2)} \tag{56}$$

$$\beta_2 = \frac{\gamma_2}{(\bar{\eta}_2^2-\bar{\eta}_1^2)(\bar{\eta}_2^2-\bar{\eta}_3^2)} \tag{57}$$

$$\beta_3 = \frac{\gamma_3}{(\bar{\eta}_3^2-\bar{\eta}_1^2)(\bar{\eta}_3^2-\bar{\eta}_2^2)} \tag{58}$$

where

$$\gamma_i = \frac{V^6}{V_1^2 V_2^2 V_3^2}[\bar\eta_i^2 \frac{V_1^2}{V^2} - \frac{V_2^2}{V^2} + 1]\{4 - [(\bar\eta_i^2 - 1)\frac{V_3^2}{V^2} + 1] - [\bar\eta_i^2 \frac{V_2^2}{V^2} - \frac{V_1^2}{V^2} + 1]\} \quad (59)$$

We take the following eigenvectors:

$$\begin{aligned} A_1^{(1)} &= -a_{22} = A_{10}^{(1)} + \hat A_1^{(1)} \frac{\Omega^2}{\omega^2} \\ A_2^{(1)} &= a_{21} \approx A_{20}^{(1)} + \hat A_2^{(1)} \frac{\Omega^2}{\omega^2}, \\ A_3^{(1)} &= -\frac{a_{31}}{a_{33}} A_1^{(1)} \approx A_{30}^{(1)} + \hat A_3^{(1)} \frac{\Omega}{\omega} \end{aligned} \quad (60)$$

$$\begin{aligned} A_1^{(2)} &= -a_{12}\bar\eta_2 \approx A_{10}^{(2)} + \hat A_1^{(2)} \frac{\Omega^2}{\omega^2} \\ A_2^{(2)} &= (a_{11} - \frac{a_{13}}{a_{12}} A_3^{(2)})\bar\eta_2 \approx A_{20}^{(2)} + \hat A_2^{(2)} \frac{\Omega^2}{\omega^2} \\ A_3^{(2)} &= -\frac{a_{31}}{a_{33}} A_1^{(2)} \bar\eta_2 \approx A_{30}^{(2)} + \hat A_3^{(2)} \frac{\Omega}{\omega} \end{aligned} \quad (61)$$

$$\begin{aligned} A_1^{(3)} &= -a_{33}/a_{31} = A_{10}^{(3)} + \hat A_1^{(3)} \frac{\Omega}{\omega} \\ A_2^{(3)} &= -\frac{a_{21}}{a_{22}} A_1^{(3)} \approx A_{20}^{(3)} + \hat A_2^{(3)} \frac{\Omega}{\omega} \\ A_3^{(3)} &= A_{30}^{(3)} + \hat A_3^{(3)} \frac{\Omega}{\omega} \end{aligned} \quad (62)$$

where

$$A_{10}^{(1)} = -(V_1^2 - V_2^2)\ ,\ A_{20}^{(1)} = (V_1^2 - V_2^2)\bar\eta_1\ ,\ A_{30}^{(1)} = 0$$

$$A_{10}^{(2)} = (V_1^2 - V_2^2)\bar\eta_2^2\ ,\ A_{20}^{(2)} = (V_2^2 - V_1^2)\bar\eta_2\ ,\ A_{30}^{(2)} = 0$$

$$A_{10}^{(3)} = 0\ , A_{20}^{(3)} = 0\ ,\ A_{30}^{(3)} = 1$$

$$\hat A_1^{(1)} = -V_1^2\beta_1,\ \hat A_2^{(1)} = (V_1^2 - V_2^2)\frac{\beta_1}{2\bar\eta_1},\ \hat A_3^{(1)} = -\frac{2V_1^2(V_1^2 - V_2^2)}{V_3^2 + V_1^2}$$

$$\hat A_1^{(2)} = \frac{(V_1^2 - V_2^2)\beta_2}{2},\ \hat A_2^{(2)} = \{V_2^2\beta_2 + V^2 - \frac{4V_2^2 V^2}{V_3^2 + V_2^2}\}\bar\eta_2,\ \hat A_3^{(2)} = \frac{2V_2^2(V_1^2 - V_2^2)}{V_3^2 + V_2^2}\bar\eta_2$$

$$\hat{A}_1^{(3)} = \frac{1}{2}\ ,\ \hat{A}_2^{(3)} = -\frac{1}{2}\frac{(V_1^2 - V_2^2)\bar{\eta}_3}{V_1^2 - V_2^2 + (1 + \frac{V_1^2}{V_3^2})V^2}\ ,\ \hat{A}_3^{(3)} = 0$$

Correspondingly, Equation (44) becomes

$$\begin{vmatrix} b_{11}^{(0)} + \hat{b}_{11}\frac{\Omega^2}{\omega^2} & b_{12}^{(0)} + \hat{b}_{12}\frac{\Omega^2}{\omega^2} & \hat{b}_{13}\frac{\Omega}{\omega} & 0 \\ b_{21}^{(0)} + \hat{b}_{21}\frac{\Omega^2}{\omega^2} & b_{22}^{(0)} + \hat{b}_{22}\frac{\Omega^2}{\omega^2} & \hat{b}_{23}\frac{\Omega}{\omega} & 0 \\ \hat{b}_{31}\frac{\Omega}{\omega} & \hat{b}_{32}\frac{\Omega}{\omega} & b_{33}^{(0)} + \hat{b}_{33}\frac{\Omega^2}{\omega^2} & b_{34}^{(0)} \\ \hat{b}_{41}\frac{\Omega}{\omega} & \hat{b}_{42}\frac{\Omega}{\omega} & b_{43}^{(0)} & b_{44}^{(0)} \end{vmatrix} = 0 \tag{63}$$

where

$$b_{1j} = A_1^{(j)}\eta_j - A_2^{(j)} \approx b_{1j}^{(0)} + \hat{b}_{1j}\frac{\Omega^2}{\omega^2}\ (j = 1, 2) \tag{64}$$

$$b_{2j} = c_{12}A_1^{(j)} + c_{11}A_2^{(j)}\eta_j \approx b_{2j}^{(0)} + \hat{b}_{2j}\frac{\Omega^2}{\omega^2}\ (j = 1, 2) \tag{65}$$

$$b_{13} = A_1^{(3)}\eta_3 - A_2^{(3)} \approx \hat{b}_{13}\frac{\Omega}{\omega}\ ,\ b_{23} = c_{12}A_1^{(3)} + c_{11}A_2^{(3)}\eta_3 \approx \hat{b}_{23}\frac{\Omega}{\omega} \tag{66}$$

$$b_{3j} = \bar{c}_{44}A_3^{(j)} = \hat{b}_{3j}\frac{\Omega}{\omega}\ ,\ b_{4j} = e_{15}A_3^{(j)} = \hat{b}_{4j}\frac{\Omega}{\omega}\ (j = 1, 2) \tag{67}$$

$$b_{33} = \bar{c}_{44}A_3^{(3)}\eta_3 \approx b_{33}^{(0)} + \hat{b}_{33}\frac{\Omega^2}{\omega^2}\ ,\ b_{34} = e_{15} = b_{34}^{(0)} \tag{68}$$

$$b_{43} = e_{15}A_3^{(3)} = b_{43}^{(0)}\ ,\ b_{44} = \epsilon_{11} = b_{44}^{(0)} \tag{69}$$

in which

$$b_{1j}^{(0)} = A_{10}^{(j)}\bar{\eta}_j - A_{20}^{(j)}\ (j = 1, 2)$$

$$b_{2j}^{(0)} = c_{12}A_{10}^{(j)} + c_{11}A_{20}^{(j)}\bar{\eta}_j\ (j = 1, 2)$$

$$b_{33}^{(0)} = \bar{c}_{44}A_{30}^{(3)}\bar{\eta}_3\ ,\ b_{43}^{(0)} = e_{15}A_{30}^{(3)}$$

$$\hat{b}_{1j} = \hat{A}_1^{(j)}\bar{\eta}_j + A_{10}^{(j)}\frac{\beta_j}{2\bar{\eta}_j} - \hat{A}_2^{(j)}\ (j = 1, 2)$$

$$\hat{b}_{2j} = c_{12}\hat{A}_1^{(j)} + c_{11}(\hat{A}_2^{(j)}\bar{\eta}_j + A_{20}^{(j)}\frac{\beta_j}{2\bar{\eta}_j})\ (j = 1, 2)$$

$$\hat{b}_{13} = \hat{A}_1^{(3)}\bar{\eta}_3 - \hat{A}_2^{(3)}\ ,\ \hat{b}_{23} = c_{12}\hat{A}_1^{(3)} + c_{11}\hat{A}_2^{(3)}\bar{\eta}_3$$

$$\hat{b}_{3j} = \bar{c}_{44}\hat{A}_3^{(j)}\ ,\ \hat{b}_{4j} = e_{15}\hat{A}_3^{(j)}\ (j = 1, 2)$$

$$\hat{b}_{33} = \bar{c}_{44}A_{30}^{(3)}\frac{\beta_3}{2\bar{\eta}_3}$$

Expanding (63) leads to

$$F(V^2, \frac{\Omega}{\omega}) = F_1(V^2)F_2(V^2) + g(V^2)\frac{\Omega^2}{\omega^2} + O(\frac{\Omega^4}{\omega^4}) = 0 \tag{70}$$

where

$$g(V^2) = \{\begin{vmatrix} b_{33}^{(0)} & b_{34}^{(0)} \\ b_{43}^{(0)} & b_{44}^{(0)} \end{vmatrix}(\begin{vmatrix} b_{11}^{(0)} & \hat{b}_{12} \\ b_{21}^{(0)} & \hat{b}_{22} \end{vmatrix} + \begin{vmatrix} \hat{b}_{11} & b_{12}^{(0)} \\ \hat{b}_{21} & b_{22}^{(0)} \end{vmatrix}) - \hat{b}_{13}b_{34}^{(0)}\begin{vmatrix} b_{21}^{(0)} & b_{22}^{(0)} \\ \hat{b}_{41} & \hat{b}_{42} \end{vmatrix}$$

$$+\hat{b}_{13}b_{44}^{(0)}\begin{vmatrix} b_{21}^{(0)} & b_{22}^{(0)} \\ \hat{b}_{31} & \hat{b}_{32} \end{vmatrix} + \hat{b}_{23}b_{34}^{(0)}\begin{vmatrix} b_{11}^{(0)} & b_{12}^{(0)} \\ \hat{b}_{41} & \hat{b}_{42} \end{vmatrix} - \hat{b}_{23}b_{44}^{(0)}\begin{vmatrix} b_{11}^{(0)} & b_{12}^{(0)} \\ \hat{b}_{31} & \hat{b}_{32} \end{vmatrix}$$

$$+\hat{b}_{33}b_{44}^{(0)}\begin{vmatrix} b_{11}^{(0)} & b_{12}^{(0)} \\ b_{21}^{(0)} & b_{22}^{(0)} \end{vmatrix}\}/\{(V_2^2 - V_1^2)^2\rho\bar{\eta}_2\} \tag{71}$$

We note that the coefficients $b_{ij}^{(0)}$ and $\hat{b}_{ij}$ depend upon the rotatoin rate Ω only through the wave speed V. We further expand $g(V^2)$, $F_1(V^2)$ and $F_2(V^2)$ near $V = \bar{V}_R$ and $V = \bar{V}_B$, respectively, and keep terms up to the second order. We obtain

$$F_1(\bar{V}_R^2)F_2(\bar{V}_R^2) + \{g(\bar{V}_R^2) + \alpha_R[F_1(\bar{V}_R^2)F_2'(\bar{V}_R^2) + F_1'(\bar{V}_R^2)F_2(\bar{V}_R^2)]\}(\frac{\Omega}{\omega})^2 = 0 \tag{72}$$

and

$$F_1(\bar{V}_B^2)F_2(\bar{V}_B^2) + \{g(\bar{V}_B^2) + \alpha_B[F_1(\bar{V}_B^2)F_2'(\bar{V}_B^2) + F_1'(\bar{V}_B^2)F_2(\bar{V}_B^2)]\}(\frac{\Omega}{\omega})^2 = 0 \tag{73}$$

We now proceed our discussion for the determination of the constants α_R and α_B by considering the following two exclusive cases:

Case I: $V_3 \leq V_2$.

In this case, we expect the absence of the Rayleigh wave at rotation if $\bar{V}_R > V_3$ because it leads to $\bar{\eta}_3^2 < 0$, according to (47). Otherwise, we note that $F_1(\bar{V}_R^2) = 0$ and obtain from (72)

$$g(\bar{V}_R^2) + \alpha_R F_1'(\bar{V}_R^2)F_2(\bar{V}_R^2) = 0 \tag{74}$$

We can determine α_R uniquely if and only if $F_1'(\bar{V}_R^2) \neq 0$ because we note from (52) that $F_2(\bar{V}_R^2) \neq 0$. Noting that $2V_2^2/3 < \bar{V}_R^2 < V_2^2 < V_1^2$, we find from (51)

$$F_1'(\bar{V}_R^2) = \frac{(V_1^2 - V_2^2)(2V_2^2 - 3\bar{V}_R^2) - 2\bar{V}_R^2(V_2^2 - \bar{V}_R^2)}{4(V_1^2 - \bar{V}_R^2)(V_2^2 - \bar{V}_R^2)} < 0 \tag{75}$$

We hence conclude the presence of the Rayleigh wave at rotation if $\bar{V}_R < V_3$.

For the Bleustein-Gulyaev wave, we note that $F_2(\bar{V}_B^2) = 0$ and obtain from (73) that

$$g(\bar{V}_B^2) + \alpha_B F_1(\bar{V}_B^2) F_2'(\bar{V}_B^2) = 0 \tag{76}$$

One can show directly from (52) that $F_2'(\bar{V}_B^2) \neq 0$. The fact that $\bar{V}_B < V_3 \leq V_2$ implies that $F_1(\bar{V}_B^2) \neq 0$ because $F_1(V^2) = 0$ has only one root $\bar{V}_R$ in the range: $(0, V_2)$. This indicates the presence of the Bleustein-Gulyaev wave at rotation.

In summary, the Bleustein-Gulyaev wave is always present, but the Rayleigh wave is present only if the Rayleigh wave speed $\bar{V}_R$ of the material in the absence of rotation is smaller than the lower shear wave speed V_3.

Case II: $V_3 > V_2$.

In this case, we can proceed our discussion similarly. We conclude that the Rayleigh wave is always present. However, the Bleustein-Gulyaev wave is present only if the Bleustein-Gulyaev wave speed $\bar{V}_B$ of the material in the absence of rotation is smaller than the lower shear wave speed V_2, i.e., $\bar{V}_B < V_2$. Otherwise, we would have $\bar{\eta}_2^2 < 0$, as for our model material PZT-5H, and the Bleustein-Gulyaev wave is absent at rotation.

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