Forced Convection in Porous Media: Transverse Heterogeneity Effects and Thermal Development

D.A. Nield and A.V. Kuznetsov

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Summary
Recent analytical studies on thermal development and transverse heterogeneity effects on forced convection in channels and ducts are surveyed.

4.1 Transverse Heterogeneity

4.1.1 Introduction
Applications of the material in this chapter include the cooling of electronic equipment using devices made of compressed metallic foam material that is highly porous even after compression. The compression may not be uniform, and the channel length may be relatively short so that the thermal development effects are important.

In this section we survey analytical studies on the effect on forced convection, in channels and ducts, of the variation in the transverse direction of permeability and thermal conductivity. Both parallel-plate channels and circular ducts are considered, and walls at uniform temperature and uniform heat flux are treated in turn. Basic work using the Darcy model for thermal equilibrium is extended in some cases to the Brinkman equation and to the case of local thermal nonequilibrium. The standard work is for symmetric property variation and symmetric thermal boundary conditions, but some exceptions are also discussed.

4.1.2 Parallel-Plate Channel
We allow the permeability $K$ and the thermal conductivity $k$ to be nonuniform in space, and define

$$\tilde{K} = \frac{K}{\bar{K}}, \quad \tilde{k} = \frac{k}{\bar{k}}$$

(4.1)

where an overbar denotes a mean value taken over the volume occupied by the porous medium.

For the steady-state fully developed situation we have unidirectional flow in the $x^*$-direction between impermeable boundaries at $y^* = -H$ and $y^* = H$, as illustrated in Figure 4.1(a). The steady-state Dupuit–Forschheimer–Brinkman equation is (for theoretical background and range of applicability, see [1])

$$G = \frac{\mu u^*}{K} + c_l \rho u^* + \mu_{eff} \frac{d^2 u^*}{dy^*}$$

(4.2)
FIGURE 4.1
Definition sketch: (a) parallel-plate channel, (b) circular duct.