Flow and heat transfer inside thin films supported by soft seals in the presence of internal and external pressure pulsations

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Abstract

The effects of both external squeezing and internal pressure pulsations are studied on flow and heat transfer inside non-isothermal and incompressible thin films supported by soft seals. The laminar governing equations are nondimensionalized and reduced to simpler forms. The upper plate displacement is related to the internal pressure through the elastic behavior of the supporting seals. The following parameters: squeezing number, squeezing frequency, frequency of pulsations, Fixation number (for the seal) and the thermal squeezing parameter are found to be the main controlling parameters. Accordingly, their influences on flow and heat transfer inside disturbed thin films are determined and discussed. It is found that an increase in the Fixation number results in more cooling and a decrease in the average temperature values. Also, it is found that an increase in the squeezing number decreases the turbulence level at the upper plate. Furthermore, fluctuations in the heat transfer and the fluid temperatures can be maximized at relatively lower frequency of internal pressure pulsations.

Keywords: Thin films; Heat transfer; Squeezing; Pressure pulsations; Seals

1. Introduction

Various engineering applications such as lubrication, heat pipes and fluidic cells of many chemical and biological detection systems require deeper understanding of flow and heat transfer inside thin films. In certain applications, external disturbances such as unbalances in rotating machines or pulsations in external ambient pressures due to many disturbances can result in an oscillatory motion at the boundary. Not only external disturbances can produce these motions, but also internal pressure pulsations such as irregularities in the pumping process can produce similar effects. Even small disturbances on the plates of the thin film can have a substantial impact as the thickness of thin films is very small. This fact is more pronounced if the thin film is supported by soft seals. Accordingly, the dynamics and thermal characterization of thin films will be altered.

The chambers for chemical and biological detection systems such as fluidic cells for chemical or biological microcantilever probes [1] are an important example for thin films. Small turbulence levels that can be introduced into these cells by either flow pulsating at the inlet or external noise that may be present at the boundaries which result in a vibrating boundary can produce flow instabilities inside the fluidic cells. These disturbances have a substantial influence on the measurements of the biological probes specially those utilizing microcantilevers as these detecting elements are very sensitive to flow conditions.

Several authors have considered flow inside squeezed thin films like Langlois [2] who performed an analytical study for flow inside isothermal oscillatory squeezed...
films with fluid density varying according to the pressure. However, only few of them have analyzed heat transfer inside squeezed thin films such as Hamza [3], Bhattacharyya et al. [4] and Debbaud [5]. In these works, the squeezing was not of oscillatory type. Recently, Khaled and Vafai [6] and [7] considered flow and heat transfer inside incompressible oscillatory squeezed thin films. The effects of internal pressure pulsations have been investigated before on flow and heat transfer inside channels (see [8,9]). However, the literature lacks an investigation of the effects of both internal and external pressure pulsations on flow and heat transfer inside thin films. In this case, the gap thickness will be a function of both pulsations.

In this work, the upper plate of a thin film is considered to be subjected to both external squeezing effects and the internal pressure pulsations. The influence of internal pressure pulsations on the displacement of the upper plate is determined by the theory of linear elasticity applied to the seal supporting the plates of an incompressible non-isothermal thin film. The laminar governing equations for flow and heat transfer are properly non-dimensionalized and reduced into simpler equations. The resulting equations are then solved numerically to determine the effects of external squeezing, internal pressure pulsations and the strength of the seal on the turbulence inside the disturbed thin films as well as on thermal characteristics of these thin films.

### 2. Problem formulation

Consider a two dimensional thin film that has a small thickness $h$ compared to its length $B$. The $x$-axis is taken in the direction of the length of the thin film while $y$-axis is taken along the thickness as shown in Fig. 1. The width of the thin film, $D$, is assumed to be large enough such that two dimensional flow inside the thin film can be assumed. The lower plate of the thin film is fixed while the vertical motion of the upper plate is assumed to have sinusoidal behavior when the thin film gap is not charged with the working fluid. This motion due to only external disturbances is expressed according to the following relation:

$$h = h_0(1 - \beta \cos(\omega t))$$  \hspace{1cm} (1)
where $\gamma$ is the dimensionless frequency and $\beta$ and $\omega$ are the dimensionless upper plate motion amplitude and a reference frequency, respectively. It is assumed that the fluid is Newtonian with constant properties.

The general two-dimensional continuity, momentum and energy equations for the laminar thin film are given as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(2)

$$\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)$$

(3)

$$\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$

(4)

$$\rho c_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

(5)

where $T$, $\rho$, $\mu$, $c_p$ and $k$ are the fluid temperature, density, pressure, dynamic viscosity, specific heat and the thermal conductivity of the fluid, respectively.

Eqs. (2)–(5) are non-dimensionalized using the following dimensionless variables:

$$X = \frac{x}{B}, \quad Y = \frac{y}{h_0}, \quad \tau = \frac{\rho \omega t}{\rho \omega + \frac{h_0}{2} \beta^2}, \quad U = \frac{u}{(\rho B + V_0)},$$

$$V = \frac{v}{h_0 \omega}, \quad \Pi = \frac{\rho - \rho_h}{\mu (\rho + \rho_h) \omega}, \quad \theta = \frac{T - T_1}{\Delta T}$$

(6)

where $T_1$ and $V_0$ are the inlet temperature of the fluid and a constant representing a reference dimensional velocity, respectively. $\Delta T$ is equal to $T_2 - T_1$ for constant wall temperature (CWT) conditions, $T_2$ will be the temperature of both lower and upper plates, and it is equal to $q h_0 / k$ for uniform wall heat flux (UHF) conditions $p_c$ is a constant representing the exit pressure. The variables $X, Y, \tau, U, V, \Pi$ and $\theta$ are the dimensionless forms of $x, y, t, u, v, p$ and $T$ variables, respectively. The above transformations except for dimensionless temperature have been used by Langlois [2] along with the perturbation parameter $\varepsilon, \varepsilon = h_0/B$.

Most flows inside thin films are laminar and could be creep flows especially in lubrications and biological applications. Therefore, the low Reynolds numbers flow model is adopted in here. The application of this model to Eqs. (2)–(5) results in the following reduced non-dimensionalized equations:

$$U = \frac{1}{2} \frac{\partial \Pi}{\partial X} (Y - H)$$

(7)

$$\frac{\partial U}{\partial X} + \frac{\sigma \partial V}{\partial Y} = 0$$

(8)

$$\frac{\partial}{\partial X} \left( H \frac{\partial \Pi}{\partial X} \right) = \frac{\partial \Pi}{\partial Y}$$

(9)

$$p_s \left( \frac{\partial \theta}{\partial \tau} + \frac{12 \sigma}{B^2} U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} \right) = \varepsilon^2 \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}$$

(10)

where $\sigma$ and $p_s$ are called the squeezing number and the thermal squeezing number, respectively. They are defined as

$$\sigma = \frac{12}{1 + \frac{h_0}{B}}, \quad p_s = \frac{\rho c_p h_0^2 \omega}{k}$$

(11)

The inlet dimensionless pulsating pressure is considered to have the following relation:

$$p_1 = p_0 (1 + \beta_p \sin(\gamma_{p \omega} t + \phi_p))$$

(12)

where $\beta_p, \Pi_1$ and $\Pi_0$ are the dimensionless amplitude in the pressure, inlet dimensionless pressure and the mean dimensionless pressure, respectively. The parameters $\gamma_p$ and $\phi_p$ are the dimensionless frequency of the pressure pulsations and a phase shift angle respectively. Due to both pulsations in internal pressure and external disturbances, the dimensionless film thickness $H$, $(H = h_0)$, can be represented by the next equation by noting the principle of superposition:

$$H = 1 - \beta \cos(\gamma t) + H_p$$

(13)

where $H_p$ is the dimensionless deformation of the seals resulting from pulsations in the internal pressure. It is assumed that the lower plate is fixed and that the upper
plate of the thin film is rigid such that the magnitude of the deformation in the seals is similar to displacement of the upper plate. The dimensionless deformation in the seals due variations in the external pressure is the second term of Eq. (13) on the right. The dimensionless frequency \( \gamma \) is allowed to be different than \( \gamma_p \).

The dimensionless pressure gradient inside the thin film as a result of the solution to the Reynolds Eq. (9) is

\[
\frac{\partial \Pi}{\partial X} = \frac{\sigma}{H^3} \frac{dH}{d\tau} \left( X - \frac{1}{2} \right) - \Pi_0(1 + \beta_p \sin(\gamma_p \tau + \varphi_p))
\]

(14)

The reference velocity, \( V_0 \), that is used to define the dimensionless pressure, axial dimensionless velocity and the squeezing number are taken to be related to the average velocity, \( u_m \), inside the thin film at zero \( \beta \) and \( \beta_q \) and the dimensionless thickness of the thin film that results from the application of the corresponding inlet mean pressure, \( H_m \), through the following relation:

\[
V_0 = \frac{u_m}{H_m}
\]

(15)

The previous scaled reference velocity is only function of the mean pressure, viscosity and the reference dimensions of the thin film and it results in the following relation between the inlet mean dimensionless pressure to the squeezing number:

\[
\Pi_0 = 12 - \sigma
\]

(16)

Accordingly, the dimensionless pressure gradient, the dimensionless pressure and the average dimensionless pressure \( \Pi_{AVG} \) inside the thin film are related to the squeezing number through the following equations:

\[
\frac{\partial \Pi(X, \tau)}{\partial X} = \frac{\sigma}{H^3} \frac{dH}{d\tau} \left( X - \frac{1}{2} \right) - (12 - \sigma) \times (1 + \beta_p \sin(\gamma_p \tau + \varphi_p))
\]

(17)

\[
\Pi(X, \tau) = \frac{\sigma}{2H^3} \frac{dH}{d\tau} (X^2 - X) - (12 - \sigma) \times (1 + \beta_p \sin(\gamma_p \tau + \varphi_p))(X - 1)
\]

(18)

\[
\Pi_{AVG}(\tau) = -\frac{\sigma}{12H^3} \frac{dH}{d\tau} \frac{1}{2} (12 - \sigma) (1 + \beta_p \sin(\gamma_p \tau + \varphi_p))
\]

(19)

The displacement of the upper plate due internal pressure pulsations is related to the \( \Pi_{AVG} \) through the theory of linear elasticity by the following relation:

\[
H_p = F_n \Pi_{AVG}
\]

(20)

where \( F_n \) is equal to

\[
F_n = \frac{\mu(V_0 + \omega B)}{E d_i}
\]

(21)

The parameters \( E \) and \( d_i \) in the previous equation are the modulus of elasticity of the used seals and a characteristic dimension for the seal, respectively. The quantity \( d_i \) is equal to the effective diameter of the seal’s cross section times the ratio of the length of the seals divided by the thin film width. The effective diameter for seals having square cross section is equal to \( h_0 \). The term \( F_n \) will be called the Fixation number of the thin film.

The Fixation number \( F_n \) represents a ratio between shear stresses inside thin films to the modulus of elasticity of the used seals. Moreover, Eq. (20) is based on the assumption that transient behavior of the seal’s deformation is negligible. The values of \( F_n \) are of order 0.001–0.1 for long thin films supported by soft seals.

The first set of dimensionless boundary conditions that will be used is for CWTs at both the lower and the upper plates while the second set is by assuming that the lower plate is at UHF conditions and the upper plate is insulated. As such the dimensionless boundary conditions can be written as

**CWT**

\[
\begin{align*}
\theta(X, Y, 0) &= 0, & \theta(0, Y, \tau) &= 0, & \theta(X, 0, \tau) &= 1 \\
\theta(X, H, \tau) &= 1, & \frac{\partial}{\partial X} \left( \frac{1 - \theta(1, Y, \tau)}{1 - \theta_m(1, \tau)} \right) &= 0
\end{align*}
\]

(22)

**UHF**

\[
\begin{align*}
\theta(X, Y, 0) &= 0, & \theta(0, Y, \tau) &= 0, & \frac{\partial \theta(0, 0, \tau)}{\partial Y} &= -1 \\
\frac{\partial \theta(H, H, \tau)}{\partial Y} &= 0, & \frac{\partial \theta(1, Y, \tau)}{\partial X} &= \frac{\sigma}{12U_m} \left( \frac{1}{P_k H} \frac{\partial \theta(1, Y, \tau)}{\partial \tau} \right)
\end{align*}
\]

(23)

The last condition of Eq. (22) is based on the assumption that the flow at the exit of the thin film is thermally fully developed. Moreover, the last thermal condition of Eq. (23) is derived based on an integral energy balance at the exit of the thin film realizing that the axial conduction is negligible at the exit. The calculated thermal parameters that will be considered are the Nusselt numbers at the lower and upper plates, and the dimensionless heat transfer from the upper and lower plates, \( \Theta \), for CWT conditions. They are defined according to the following equations:

**CWT**

\[
\begin{align*}
Nu_u(X, \tau) &= \frac{h_u h_0}{k} \left( 1 - \frac{\partial \theta(X, H, \tau)}{\partial Y} \right) \\
Nu_l(X, \tau) &= \frac{h_l h_0}{k} \left( -\frac{1}{1 - \theta_m(X, \tau)} \frac{\partial \theta(0, X, \tau)}{\partial Y} \right)
\end{align*}
\]

(24)

\[
\Theta(X, \tau) = \left( \frac{\partial \theta(X, H, \tau)}{\partial Y} - \frac{\partial \theta(X, 0, \tau)}{\partial Y} \right)
\]

**UHF**

\[
\begin{align*}
Nu_u(X, \tau) &= \frac{h_u h_0}{k} \left( 1 - \frac{\theta(X, 0, \tau) - \theta_m(X, \tau)}{1 - \theta_m(X, \tau)} \right)
\end{align*}
\]

(25)
where $\theta_m$ and $U_m$ are the dimensionless mean bulk temperature and the dimensionless average velocity at a given section. They are defined as follows:

$$\theta_m(X, \tau) = \frac{1}{U_m(X, \tau)H} \int_0^H U(X, Y, \tau) \theta(X, Y, \tau) dY$$

$$U_m(X, \tau) = \frac{1}{H} \int_0^H U(X, Y, \tau) dY$$

(26)

Due to symmetric flow and thermal conditions for CWT, it is expected that Nusselt numbers at lower and upper plates to be equal.

3. Numerical methods

The dimensionless thickness of the thin film was determined by solving Eqs. (19), (20) and (13) simultaneously. Accordingly, the velocity field, $U$ and $V$, was determined from Eqs. (7) and (8). The reduced energy equation, Eq. (10), was then solved using the alternative direction implicit (ADI) techniques by transferring the problem to one with constant boundaries using the following transformations: $\tau' = \tau$, $\zeta = X$ and $\eta = Y/H$. Iterative solution was employed for the $\zeta$-sweep of the energy equation for CWT conditions so that both the energy equation and the exit thermal condition, last condition of Eq. (22), are satisfied. The values of 0.008, 0.03, 0.002 were chosen for $D_{n}$, $D_{g}$ and $D_{s}/C_3$.

4. Discussions of the results

4.1. Effects of pressure pulsations on the dimensionless film thickness

Figs. 2 and 3 describe the importance of the Fixation number $F_n$ on the dimensionless film thickness $H$ and the dimensionless normal velocity at the upper plate $V(X, H, \tau)$, respectively. It is noticed that as $F_n$ increases, $H$ and absolute values of $V(X, H, \tau)$ increase. It is worth noting that Soft fixations have large $F_n$ values. Increases in the viscosity and flow velocities or a decrease in the thin film thickness, perturbation parameter and the seal’s modulus of elasticity increase the value of $F_n$ as Eq. (21) predicts.

The effects of pressure pulsations on $H$ are clearly seen for large values of $F_n$ as shown in Figs. 2 and 3. At these values, the frequency of the local maximum or minimum of $H$ is similar to the frequency of the pressure pulsations as seen from Fig. 2. Further, the degree of turbulence at the upper plate is increased when $F_n$ increases as shown in Fig. 3. The fluctuations and the number of local maximum and minimum in $V(X, H, \tau)$ are meant by the degree of turbulence at the upper plate. This is also obvious when the values of $\gamma_p$ increase as shown in Fig. 4. The increase in turbulence level at the upper plate may produce back flows inside the thin film at large values of $\gamma_p$. This affects the function of the thin film especially that used as a chamber for detection purposes.

For $\sigma = 12$ where the time average of the average gage pressure inside the thin film is zero, the variation in
$H$ decreases as $F_n$ increases. This effect can be seen from Eqs. (19) and (20) and will cause reductions in the flow and in the cooling process. However, the mean value of $H_{\text{AVG}}$ is always greater than zero for other values of $\sigma$ which causes an increase in the mean value of $H$ as $F_n$ increases resulting in an increase in the mean value of the flow rate inside the thin film.

Fig. 5 shows the effects of the squeezing number $\sigma$ on $H$. Small values of $\sigma$ indicates that the thin film is having relatively large inlet flow velocities thus it has large pressure gradients and large values of $H_{AVG}$. Accordingly, $H$ increases as $\sigma$ decreases as seen in Fig. 5. Further, it is noticed that the degree of turbulence at the upper plate increases as $\sigma$ decreases. This is shown in Fig. 6. The changes in the pressure phase shift results in similar changes in the dimensionless thin film thickness phase shift as shown in Fig. 7.
4.2. Effects of pressure pulsations on heat transfer characteristics of the thin film

Figs. 8 and 9 illustrate the effects of $F_n$ and $P_S$ on the dimensionless mean bulk temperature $\theta_m$ and the average lower plate temperature $\theta_W$, average of $\theta(X, 0, \tau)$, for CWT and uniform heat flux UHF conditions, respectively. As $F_n$ increases when softer seals are used, the induced pressure forces inside the thin film due to internal pressure pulsations will increase the displacement of the upper plate as shown before. This enables the thin film to receive larger flow rates since all the cases presented in these figures have similar values for the dimensionless pressure at the inlet. Thus, more cooling to the plates results as $F_n$ increases resulting in a decrease in the $\theta_m$ and average $\theta_W$ values and their corresponding fluctuations for CWT and UHF conditions, respectively.

The effect of the thermal squeezing parameter $P_S$ on the cooling process is also shown in Figs. 8 and 9. It is

![Fig. 8. Effects of $F_n$ and $P_S$ on $\theta_m$ (CWT).](image1)

![Fig. 9. Effects of $F_n$ and $P_S$ on $\theta_W$ (UHF).](image2)

![Fig. 10. Effects of $F_n$ on $Nu_L$ (CWT).](image3)

![Fig. 11. Effects of $F_n$ on $Nu_L$ (UHF).](image4)
shown that the cooling at the plates is enhanced as $P_s$ increases.

Figs. 10 and 11 show the effects of $F_n$ on the Nusselt number at the lower plate $Nu_L$ for CWT and UHF conditions, respectively. It is noticed that the irregularity in $Nu_L$ decrease as $F_n$ decreases. This is because the upper plate will not be affected by the turbulence in the flow if the used seals have relatively large modulus of elasticity. In other word, the induced flow due to the upper plate motion is reduced as $F_n$ decreases resulting in less disturbances to the flow inside the thin film. This can be seen in Fig. 12 for UHF conditions where $Nu_L$ reaches a constant value at low values of $F_n$ after a certain distance from the inlet. The values of $Nu_L$ and the corresponding fluctuations are noticed to decrease as $F_n$ increases.

Figs. 13 and 14 illustrate the effects of dimensionless frequency of the inlet pressure pulsations $\gamma_p$ on the average dimensionless heat transferred from the plates $\Theta$ and the average $\theta_w$ for CWT and UHF conditions, respectively. The figures show that the mean value of $\Theta$
and $\theta_w$ are unaffected by $\gamma_p$ and that the frequency of the average values of $\Theta$ and $\theta_w$ increase as $\gamma_p$ increases. Fig. 15 describes the effects of $\gamma_p$ on the fluctuation in the average $\Theta$ and $\theta_w$, half the difference between the maximum and the minimum values of the average $\Theta$ and $\theta_w$. The effects of $\gamma_p$ on the fluctuation in the average $\Theta$, $\Delta\Theta$, and the fluctuation in the average $\theta_w$, $\Delta\theta_w$, are more pronounced at relatively lower values of $\gamma_p$.

5. Conclusions

Flow and heat transfer inside externally oscillatory squeezed thin film supported by soft seals in the presence of inlet internal pressure pulsations have been analyzed in this work. The governing laminar continuity, momentum and energy equations were properly non-dimensionalized and reduced to simpler forms for small Reynolds numbers. The reduced equations were solved by the ADI method. It was found that the turbulence level at the upper plate increases by increases in both the Fixation number and the frequency of the internal pressure pulsations. However, an increase in the squeezing number decreases the turbulence level at the upper plate. The fluid temperatures and the corresponding fluctuations were found to decrease when the Fixation number and the thermal squeezing parameter were increased for both CWT and UHF conditions. Finally, fluctuations in the heat transfer and the fluid temperatures are more pronounced at relatively lower frequency of internal pressure pulsations. This study can lead the way for further investigations on the effects of internal pressure pulsations on thin films having relatively large Reynolds numbers.

References